Application of the CFS PML to the Absorption of Evanescent Waves in Waveguides

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***Abstract*—This letter shows that the complex frequency shifted PML (CFS PML) can be optimized in view of absorbing both the evanescent and traveling waves at the end of waveguiding structures. Numerical results are provided to illustrate the effectiveness of the optimized CFS PML in FDTD computations.**

***Index Terms*—Absorbing boundary condition, FDTD, PML, waveguide.**

I. Introduction

SINCE the introduction of the PML concept in numerical electromagnetics, the absorption of evanescent waves at the end of waveguides terminated by PMLs has been addressed in several papers [1]–[4]. In such problems, the evanescent waves are not absorbed by the PML because the evanescence is perpendicular to the vacuum-PML interface, i.e., the propagation is parallel to the PML. The first attempt to overcome this drawback has been the use of a real stretch of coordinates within the PML [2], [3], so as to enhance the natural decrease of the evanescent fields. Another and better solution consists of using the modified PML [4] that allows a quite good absorption of both evanescent and traveling waves.

In this letter, we show that the CFS PML [5] could also be an effective solution for terminating waveguides in numerical electromagnetics. It is shown that, by an adequate choice of its characteristic parameter α_x, the CFS PML also allows both the evanescent and traveling waves to be annihilated. An important advantage of the CFS PML in comparison with the PML-D [4] is its simple FDTD implementation [6], [7].

II. The CFS PML

The CFS PML was introduced by Kuzuoglu and Mittra [5] to render the PML causal. To this end, the stretching coefficient of the normal PML is replaced by

$$s_x = \kappa_x + \frac{\sigma_x}{\alpha_x + j\omega\varepsilon} \tag{1}$$

where α_x is homogeneous to a conductivity and κ_x is real. In this letter, we assume that $\kappa_x = 1$. An effective FDTD discretization has been proposed by Roden and Gedney [6]. The theoretical features and the numerical reflection of this PML have been derived and discussed in [8]. From (1), it is evident that frequency

$$f_\alpha = \frac{\alpha_x}{2\pi\varepsilon} \tag{2}$$

is a key parameter. For $f \gg f_\alpha$, the CFS PML acts as a normal PML. Conversely, for $f \ll f_\alpha$, it does not absorb the waves and only acts as a real stretch of coordinates $1+\sigma_x/\alpha_x$. Derivations in [8] show that evanescent waves can exist in the CFS PML. By denoting as $\cosh\chi$ the coefficient of evanescence, θ the direction of propagation, and ψ_{vacuum} the waveform in a vacuum, such evanescent waves can be written as [8]

$$|\psi_{PML}| = |\psi_{vacuum}| e^{-(\sigma_x/\varepsilon c)\, \cosh\chi \cos\theta x} \quad \text{for } f \gg f_\alpha \tag{3}$$

$$|\psi_{PML}| = |\psi_{vacuum}| e^{(f/f_\alpha)\,(\sigma_x/\varepsilon c)\, \sinh\chi \sin\theta x} \quad \text{for } f \ll f_\alpha. \tag{4}$$

As expected, for $f \gg f_\alpha$, the absorption is like in a usual PML [9], and for $f \ll f_\alpha$, the traveling waves $(\sinh\chi = 0)$ are not absorbed. For a wave evanescent toward $+x$, the real exponential in (4) is an absorption, since then θ is negative, due to the choice of the signs in the waveform [8]. Thus, for $f \ll f_\alpha$, the evanescent waves are absorbed by the CFS PML. By combining the coefficient in (4) with the natural decrease in ψ_{vacuum}, it can be shown that the overall decrease equals the natural decrease upon distance $(1 + \sigma_x/\alpha_x)x$, in accordance with the low frequency limit of (1).

We can notice that the absorption of evanescent waves (4) depends on frequency, so that one could fear that wide-band applications could not be achieved. Fortunately, f and $\sinh\chi$ depend on each other in a favorable manner in waveguides. More precisely, the product $f \sinh\chi$ is about constant below the cutoff frequency. This will permit a reasonable absorption of evanescent waves at any frequency below this cutoff.

III. Application to the Termination of Waveguides

Let us now consider the absorption of the TM$_{n0}$ mode at the end of a waveguide of size a. For this mode, the dependence of the evanescent waves in the longitudinal direction x is of the form

$$\exp\left(-\frac{\omega}{c}\, \sinh\chi x\right)$$

where

$$\sinh\chi = \sqrt{\frac{n^2\pi^2c^2}{a^2\omega^2} - 1}. \tag{5}$$

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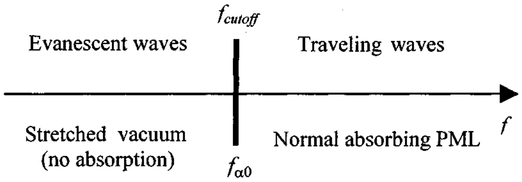

Fig. 1. Coincidence of frequency $f_{\alpha 0}$ with the waveguide cutoff.

From this, far below the cutoff frequency, the following relationship holds

$$\omega \sinh \chi = \frac{n\pi c}{a} \qquad \text{for } \omega \ll \frac{n\pi c}{a}. \tag{6}$$

Let now a waveguide be terminated by a CFS PML. This requires the parameter α_x to be set. The absorption of evanescent waves depends on α_x through f_α in (4). Ideally, this absorption should be both sufficient and not too strong to avoid a strong numerical reflection [9]. This can be achieved by equating the absorbing coefficient of evanescent waves in (4) to the absorption of traveling waves at normal incidence in (3). This yields

$$-\frac{\sigma_x}{\varepsilon c} = \frac{\omega \varepsilon}{\alpha_x}\frac{\sigma_x}{\varepsilon c}\sinh \chi \sin \theta. \tag{7}$$

With (6), and since $\theta = -\pi/2$ for the evanescent waves, the optimum α_x is then

$$\alpha_0 = n\frac{\pi c \varepsilon}{a}. \tag{8}$$

From (2), frequency f_α corresponding to α_0 is

$$f_{\alpha 0} = \frac{nc}{2a} = f_{cutoff}. \tag{9}$$

Thus, with α_x (8), the frequency f_α of the transition between the two regimes of the CFS PML is also the frequency f_{cutoff} of the transition between the evanescent and traveling waves of the considered mode in the waveguide. This is summarized in Fig. 1.

IV. Numerical Results

Results are provided in Figs. 2–5 to illustrate the effectiveness of the CFS PML when α_x equals its optimum value (8). These results were computed by the theory of the numerical reflection [8]. The result of a FDTD experiment is provided in Fig. 2 to validate the theory. All the results are for the TM_{10} mode of the two-dimensional parallel-plate previously considered in papers [3] and [4] (40-mm thick, $\Delta x = 1$ mm, cutoff 3.75 GHz).

Fig. 2 shows the reflection coefficient in function of frequency for an eight cell thick PML having a parabolic profile of conductivity and a normal absorption of 200 dB. Results are provided for a normal split PML and for CFS PMLs whose α_x parameters are around the optimum value (8). The improvement resulting from the replacement of the normal PML by a CFS PML of α_x close to α_0 is dramatic in the evanescent region. Only a band of frequency is poorly absorbed around the waveguide cutoff. This is because the theoretical absorption vanishes as the frequency tends to the cutoff. Notice the perfect agreement of the FDTD experiment with the theory

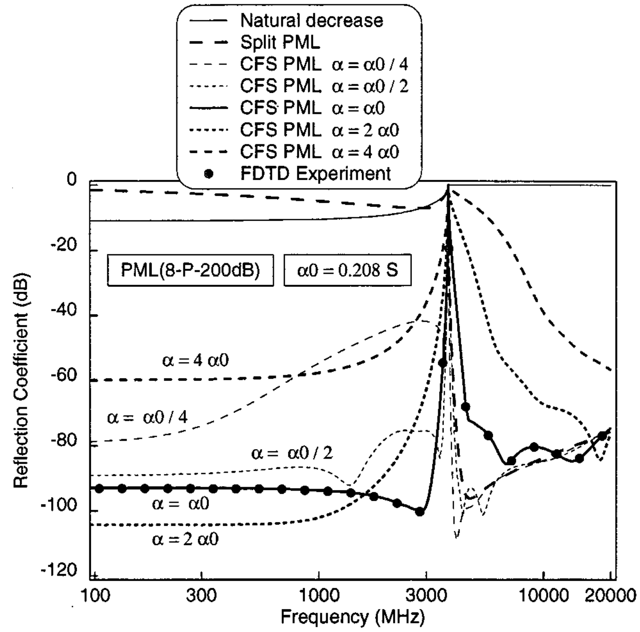

Fig. 2. Reflection of the TM_{10} mode from a normal split PML and various CFS PMLs.

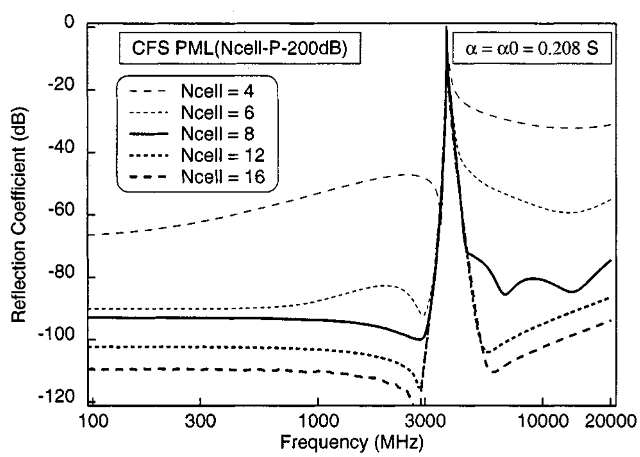

Fig. 3. Effect of the thickness $Ncell$ of the CFS PML.

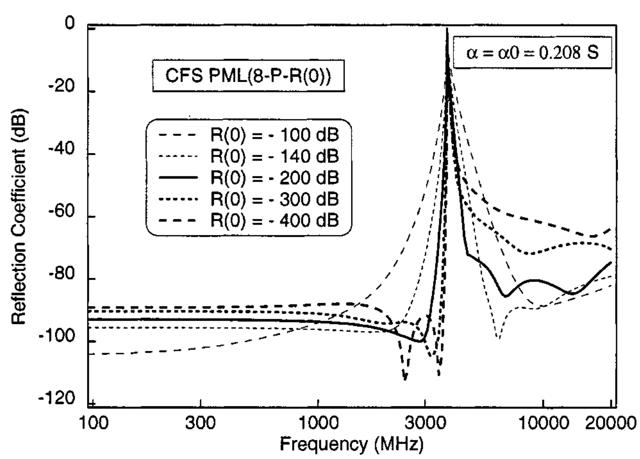

Fig. 4. Effect of the reflection $R(0)$ of the CFS PML.

of the numerical reflection for $\alpha_x = \alpha_0$. Such an agreement is

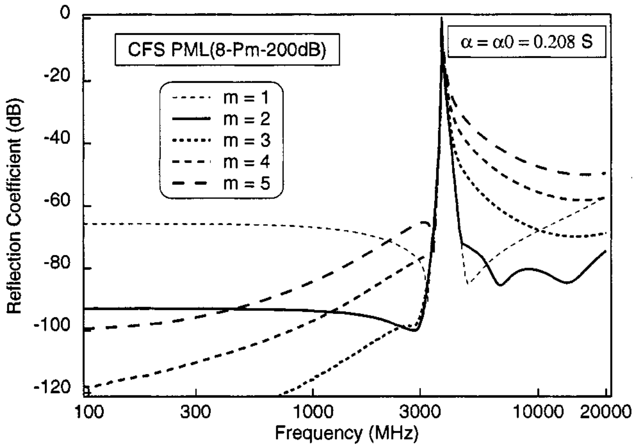

Fig. 5. Effect of the degree m of the polynomial profile of conductivity.

general, it has been observed with all the CFS PMLs considered in this letter.

Fig. 3 shows the effect of the PML thickness. The thicker the CFS PML the better the absorption of evanescent and traveling waves. But, the width of the peak of reflection around the cutoff frequency does not depend on the thickness.

Fig. 4 shows the effect of the normal reflection $R(0)$ of the CFS PML. As observed, the stronger the absorption the narrower the reflection around the cutoff frequency. This is because the theoretical absorption decreases on both sides of the cutoff, so that the band of frequency in which the absorption is small is narrowed by decreasing $R(0)$. Nevertheless, the value of $R(0)$ is limited since a too small $R(0)$ results in numerical reflection far from the cutoff frequency.

Finally, Fig. 5 illustrates the effect of the profile of conductivity for polynomial profiles of the form $\sigma(x) = \sigma_{\max}(x/d)^m$, where d is the PML thickness. The best results are obtained with $m = 2$ (parabolic profile) in the traveling region, and $m = 3$ in the evanescent region. In fact, the optimum *m* slightly depends on the PML. As an example, with a 12-cell PML, $m = 3$ yields the best results in the two regions. In practice, $m = 2$ or 3 always allows a very good absorption of both traveling and evanescent waves.

V. CONCLUSION

At low frequency, the CFS PML [5] acts as a real stretch of coordinates, instead of absorbing the waves. This is not a drawback but an advantage for annihilating evanescent fields. By an adequate choice of the characteristic parameter α_x, both traveling and evanescent waves can be absorbed at the end of waveguides. Some advantages of the CFS PML are, first, the strong absorption that can be realized with PMLs of only 8–12 cells in thickness and, second, the easy and effective FDTD implementation [7]. A drawback may be the fact that the parameter α_x can be optimized for only one mode of the waveguide.

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